



## *Heat, State Change, Calorimetry*

### Solutions to “Self Test Problems”

### 2013 Printing (see Note\*)

**Note\*:** The only difference between these solutions and those in the 2012 printing and earlier is that here the value used for the specific heat of water is 4.183 kJ/kg. In some questions this was previously stated as 4.2 kJ/kg. While the answers are almost insignificantly different, nevertheless it was felt important to remain consistent in all questions and with the 4.183 kJ/kg as stated in the Academic Supplement.

The questions affected are numbers 5, 6, 7, 9, 10, 11, 12, 13, 16, and 17

1. A solid, having a mass of 750 g, has 15.3 kJ of heat transferred to it, causing its temperature to increase to 38°C from 14°C. Calculate the specific heat of the material from which the solid is made.

$$\begin{aligned} Q &= m C_p (T_2 - T_1) \\ \therefore C_p &= \frac{Q}{m(T_2 - T_1)} \\ &= \frac{15.3 \text{ kJ}}{0.75 \text{ kg} (38 - 14)} \\ &= \frac{15.3}{18} \text{ kJ/kg } ^\circ\text{K} \\ &= \mathbf{0.85 \text{ kJ/kg } ^\circ\text{K (Ans.)}} \end{aligned}$$

2. One cubic metre of mercury has 28.356 MJ of heat transferred from it. Calculate the temperature change, in °C. Mercury has a density of 13 600 kg/m<sup>3</sup> and its specific heat is 0.139 kJ/kgK.

$$\begin{aligned} Q &= m C_p (T_2 - T_1) \\ \therefore T_2 - T_1 &= \frac{Q}{m C_p} \\ &= \frac{28\,356 \text{ kJ}}{13\,600 \text{ kg} \times 0.139 \text{ kJ/kg } ^\circ\text{K}} \\ &= 15^\circ\text{C} \end{aligned}$$

**The temperature drops by 15°C. (ie. a temperature change of - 15°) (Ans.)**

3. A material has a specific heat of 0.414 kJ/kgK. What is the mass of the material if 16.5 MJ of heat, added or removed, causes the temperature to change by 19°C?

$$Q = m C_p (T_2 - T_1)$$

$$\begin{aligned} \therefore m &= \frac{Q}{C_p (T_2 - T_1)} \\ &= \frac{16\,500 \text{ kJ}}{0.414 \text{ kJ/kg}^\circ\text{K} \times 19} \\ &= \mathbf{2098 \text{ kg (Ans.)}} \end{aligned}$$

4. An 85 kg block of ice at  $-25^\circ\text{C}$  receives 1600 kJ of heat. A second block of ice, at  $-1^\circ\text{C}$  and 58 kg, has 2100 kJ of heat removed. Which block ends up at the coldest temperature and by how much is it colder? Specific heat of ice = 2.135 kJ/kgK.

Consider the 85 kg block:

$$T_2 - T_1 = \frac{Q}{m C_p}$$

$$T_2 - (-25^\circ\text{C}) = \frac{1600 \text{ kJ}}{85 \times 2.135 \text{ kJ/kg}^\circ\text{K}}$$

$$T_2 + 25^\circ\text{C} = 8.82$$

$$\therefore T_2 = 8.82 - 25 = -16.18^\circ\text{C}$$

Consider the 58 kg block:

$$T_2 - T_1 = \frac{Q}{m C_p}$$

$$T_2 - (-1^\circ\text{C}) = \frac{-2100 \text{ kJ}}{58 \text{ kg} \times 2.135 \text{ kJ/kg}^\circ\text{K}}$$

$$T_2 + 1 = -16.96$$

$$T_2 = -1 - 16.96 = -17.96$$

$\therefore$  **The 58 kg block becomes colder than the 85 kg block by  $19.96 - 16.18 = 1.78^\circ$  (Ans.)**

5. 500 kg of steam at atmospheric pressure and  $100^\circ\text{C}$  are condensed to a final temperature of  $65^\circ\text{C}$ . How much heat is removed from the steam by the condenser?

Total heat removed = latent heat + sensible heat

$$= 500 \text{ kg} \times 2257 \text{ kJ/kg} + 500 \text{ kg} \times 4.183 \text{ kJ/kg}^\circ\text{C} \times (100 - 65)^\circ\text{C}$$

$$= 1\,128\,500 \text{ kJ} + 73\,202.5 \text{ kJ}$$

$$= 1\,201\,702.5 \text{ kJ}$$

$$= \mathbf{1201.7 \text{ MJ (Ans.)}}$$

6. 150 kg of ice at  $-20^{\circ}\text{C}$  is converted to water at  $100^{\circ}\text{C}$ . How much heat is absorbed by each kg during this change?

Sensible heat to raise 1 kg ice to  $0^{\circ}\text{C}$ :

$$\begin{aligned} &= m C_p (T_2 - T_1) \\ &= 1 \times 2.135 \times [0 - (-20)] \\ &= 42.7 \text{ kJ} \end{aligned}$$

Latent heat to change 1 kg ice to water:

$$= 335 \text{ kJ}$$

Sensible heat to raise 1 kg water to  $100^{\circ}\text{C}$ :

$$\begin{aligned} &= m C_p (T_2 - T_1) \\ &= 1 \times 4.183 \times 100 \\ &= 418.3 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \therefore \text{Heat absorbed by each kg} \\ &= 42.7 + 335 + 418.3 \text{ kJ} \\ &= \mathbf{796 \text{ kJ (Ans.)}} \end{aligned}$$

7. Enough heat is added to 1400 kg of ice, at  $-12^{\circ}\text{C}$ , to convert 900 kg to water at  $75^{\circ}\text{C}$  (at which point it is taken for process purposes) and the remainder to steam at  $100^{\circ}\text{C}$ . How many gigajoules of heat are required to do this?

Heat required to raise 1400 kg ice to  $0^{\circ}\text{C}$ :

$$\begin{aligned} &= 1400 \times 2.135 \times [0 - (-12)] \\ &= 1400 \times 2.135 \times 12 \\ &= 35\,868 \text{ J} \end{aligned}$$

Heat required to change 1400 kg ice to water:

$$\begin{aligned} &= 1400 \times 335 \\ &= 469\,000 \text{ kJ} \end{aligned}$$

Heat required to raise 900 kg water to  $75^{\circ}\text{C}$ :

$$\begin{aligned} &= 900 \times 4.183 \times (75 - 0) \\ &= 282\,352.5 \text{ kJ} \end{aligned}$$

Heat required to change 500 kg water to steam at  $100^{\circ}\text{C}$ :

$$\begin{aligned} &= 500 \times 4.183 \times (100 - 0) + 500 \times 2257 \\ &= 209\,150 + 1\,128\,500 \text{ kJ} \\ &= 1\,337\,650 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \therefore \text{Total heat required:} \\ &= 35\,868 + 469\,000 + 282\,352.5 + 1\,337\,650 \text{ kJ} \\ &= 2\,124\,870.5 \text{ kJ} \\ &= \mathbf{2.125 \text{ GJ (Ans.)}} \end{aligned}$$

8. Which of the following involves the greatest amount of heat and by how much:

- a) melting 1100 kg of copper, originally at 60°C, or  
 b) vaporizing 28 kg of silver at the evaporation temperature?

Use tables in the module for the necessary figures.

a) Consider the copper:

$$\text{Melting temperature} = 1083^\circ\text{C}$$

$$\text{Specific heat} = 0.388 \text{ kJ/kg}$$

$$\text{Latent heat of fusion} = 134 \text{ kJ/kg}$$

∴ Heat required to melt the copper:

$$= 1100 \text{ kg} \times 0.388 \times (1083 - 60) + 1100 \times 134$$

$$= 436\,616.4 + 147\,400 \text{ kJ}$$

$$= 584\,016.4 \text{ kJ}$$

b) Consider the silver:

$$\text{Latent heat of vaporization} = 20\,335 \text{ kJ/kg}$$

∴ Heat required to vaporize the silver:

$$= 28 \text{ kg} \times 20\,335 \text{ kJ/kg}$$

$$= 569\,380 \text{ kJ}$$

∴ **Melting the copper requires** (584 016.4 – 569 380)

$$= \mathbf{14.64 \text{ MJ more heat than vaporizing the silver. (Ans.)}$$

9. Steam at atmospheric pressure and 373 K, having a mass of 3.6 kg, is condensed. The water is cooled to 0°C and changed to ice at 273 K. If the ice is then cooled to –23°C, what is the total megajoules of heat removed?

$$373 \text{ K} = 100^\circ\text{C}; 273 \text{ K} = 0^\circ\text{C}$$

For each kg of steam the heat removed is:

$$\text{Latent heat of evaporation} = 2257 \text{ kJ/kg}$$

Sensible heat to reduce water to 0°C

$$= 4.183 \text{ kJ} \times 100$$

$$= 418.3 \text{ kJ/kg}$$

$$\text{Latent heat of fusion to ice} = 335 \text{ kJ/kg}$$

Sensible heat to reduce ice to –23°C:

$$= 2.135 \times 23$$

$$= 49.1 \text{ kJ/kg}$$

∴ Heat removed per kg = 2257 + 418.3 + 335 + 49.1 kJ

$$= 3059.4 \text{ kJ}$$

So, Heat removed from 3.6 kg = 3.6 × 3059.4 kJ

$$= 11\,013.84 \text{ kJ}$$

$$= \mathbf{11.014 \text{ MJ (Ans.)}$$

10. What will the temperature of the water be, at thermal equilibrium, if 130 kg of copper, at 700°C is dropped into 600 litres of water at 20°C. The specific heat of copper is 0.388 kJ/kgK and specific heat of water = 4.183 kJ/kgK.

The water and copper will both have a final temperature,  $t_2$ .

$$\begin{aligned}\text{Heat gained by water} &= m c \Delta t \\ &= 600 \times 4.183 \times (t_2 - 20)\end{aligned}$$

$$\begin{aligned}\text{Heat lost by copper} &= m c \Delta t \\ &= 130 \times 0.388 \times (700 - t_2)\end{aligned}$$

Heat gained by water = heat lost by copper

$$\therefore 600 \times 4.183 \times (t_2 - 20) = 130 \times 0.388 \times (700 - t_2)$$

$$2509.8 t_2 - 50\,196 = 35\,308 - 50.44 t_2$$

$$2560.24 t_2 = 85\,504$$

$$t_2 = \frac{85\,504}{2560.24} = \mathbf{33.4^\circ\text{C (Ans.)}}$$

11. Find the final temperature if 400 kg of ice at 0°C are mixed with 90 kg of steam at 100°C, at atmospheric pressure. Use specific heat of water as 4.183 kJ/kgK.

Let the final mixture temperature =  $t$

Heat gained by ice = heat lost by the steam

Heat gained by ice = latent heat of fusion + sensible heat

$$\begin{aligned}&= 400 \text{ kg} \times 335 \text{ kJ/kg} + 400 \text{ kg} \times 4.183 \times (t - 0) \\ &= 134\,000 + 1673.2 t\end{aligned}$$

Heat lost by ice = latent heat of vapour + sensible heat

$$\begin{aligned}&= 90 \times 2257 + 90 \times 4.183 \times (100 - t) \\ &= 203\,130 + 37\,647 - 376.47 t \\ &= 240\,777 - 376.47 t\end{aligned}$$

$$\therefore 134\,000 + 1673.2 t = 240\,777 - 376.47 t$$

$$1673.2 t + 376.47 t = 240\,777 - 134\,000$$

$$2049.7 t = 106\,777$$

$$\therefore t = \frac{106\,777}{2049.7} = \mathbf{52.1^\circ\text{C (Ans.)}}$$

12. Assuming no heat loss to atmosphere, what is the final temperature if 15 tonnes of water, at 25°C, are injected with 250 kg of steam, at atmospheric pressure and 100°C?

Let final temperature =  $t$

Heat gained by water = heat lost by steam

$$15\,000 \text{ kg} \times 4.183 \times (t - 25) = 250 \times 2257 + 250 \times 4.183 \times (100 - t)$$

$$62\,745 t - 1\,568\,625 = 564\,250 + 104\,575 - 1045.75 t$$

$$62\,745 t + 1045.75 t = 564\,250 + 104\,575 + 1\,568\,625$$

$$63\,790.75 t = 2\,237\,450$$

$$t = \frac{2\,237\,450}{63\,790.75} = \mathbf{35.07^\circ\text{ (Ans.)}}$$



13. A 2 tonne block of ice, at 18°C below the freezing point, is melted and totally converted to water, at 4°C, by mixing it with steam at atmospheric pressure and 100°C. How much steam, in kg, is required, assuming no heat losses to atmosphere and all the steam becomes water at the final mixture temperature?

Heat required to change the ice to water at 4°C:

$$\begin{aligned} &= \text{sensible heat} + \text{latent heat} + \text{sensible heat} \\ &= 2000 \text{ kg} (2.135 \times 18 + 335 + 4.183 \times 4) \text{ kJ/kg} \\ &= 2000 (38.43 + 335 + 16.73) \text{ kJ} \\ &= 780\,320 \text{ kJ} \end{aligned}$$

∴ Heat required from the steam = 780 320 kJ.

Heat removed to convert 1 kg steam to water at 4°C:

$$\begin{aligned} &= \text{latent heat} + \text{sensible heat} \\ &= 2257 \text{ kJ} + 4.183 \times (100 - 4) \text{ kJ} \\ &= 2257 + 401.57 \text{ kJ} \\ &= 2658.57 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \therefore \text{kg of steam required} &= \frac{780\,320 \text{ kJ}}{2658.57 \text{ kJ/kg}} \\ &= \mathbf{293.51 \text{ kg (Ans.)}} \end{aligned}$$

14. The data from a calorimeter test, involving an aluminum calorimeter, is as follows:

- mass of tested material = 500g
- mass of calorimeter = 0.95 kg
- mass of water = 1.2 kg
- initial temperature of tested material = 80°C
- initial temperature of calorimeter = 10°C
- temperature change of the calorimeter = 6.5°C

What is the specific heat of the tested material? Use 4.183 kJ/kgK for the specific heat of water.

Given: final temperature = 10°C + 6.5°C = 16.5°

Heat lost by material = heat gain by water + heat gain by aluminum

Let  $c_M$  = specific heat of the material tested

$$m c \Delta T_M = m c \Delta T_W + m c \Delta T_A$$

$$0.5 \times c_M \times (80 - 16.5) = 1.2 \times 4.183 \times 6.5 + 0.95 \times 0.909 \times 6.5$$

$$31.75 c_M = 32.63 + 5.61$$

$$\therefore c_M = \frac{38.24}{31.75} = \mathbf{1.204 \text{ kJ/kgK (Ans.)}}$$

15. A copper calorimeter has a mass of 0.60 kg and contains 650 g of water. The water and copper are heated to 30°C and 1.0 kg of a substance is placed in the calorimeter. If the substance was initially at 92°C and was discovered to have a specific heat of 0.337 kJ/kgK, what was the final temperature of the calorimeter?

$$m c \Delta T_s = m c \Delta T_w + m c \Delta T_c$$

$$c \text{ for copper} = 0.388$$

Let final temperature = t

$$1 \times 0.337 \times (92 - t) = 0.65 \times 4.183 \times (t - 30) + 0.6 \times 0.388 \times (t - 30)$$

$$31.004 - 0.337 t = 2.719 t - 81.569 + 0.233 t - 6.984$$

$$-0.337 t - 2.719 t - 0.233 t = 81.569 - 6.984 - 31.004$$

$$-3.289 t = -119.557$$

$$\therefore t = \frac{-119.557}{-3.289} = \mathbf{36.35^\circ\text{C (Ans.)}}$$

16. The material of a calorimeter has a specific heat of 0.467 kJ/kgK and a mass of 0.6 kg. What is its water equivalent? Use 4.183 kJ/kgK for specific heat of water.

$$\begin{aligned} \text{mass}_{(\text{water})} &= \frac{\text{mass}_{(\text{calorimeter})} \times C_{(\text{calorimeter})}}{C_{(\text{water})}} \\ &= \frac{0.6 \text{ kg} \times 0.467 \text{ kJ/kgK}}{4.183 \text{ kJ/kgK}} \\ &= \mathbf{0.067 \text{ kg (Ans.)}} \end{aligned}$$

17. What is the mass of a material that has a water equivalent of 7.45 kg and a specific heat of 0.875 kJ/kgK?

$$\begin{aligned} \text{mass of material} &= \frac{\text{water equivalent} \times C_{(\text{water})}}{C_{(\text{material})}} \\ &= \frac{7.45 \text{ kg} \times 4.183 \text{ kJ/kgK}}{0.875 \text{ kJ/kgK}} \\ &= \mathbf{35.62 \text{ kg (Ans.)}} \end{aligned}$$



18. A copper calorimeter has a mass of 0.5 kg. If it contains 0.6 kg of water at 22°C when a test material is placed in it,

- what is the water equivalent of the calorimeter, and
- how much total heat must the calorimeter + water absorb if equilibrium temp. is 31°C?

The test determines the specific heat of the material to be 0.435 kJ/kgK.

a) Water equivalent of the calorimeter:

$$\begin{aligned}\text{Water equivalent}_{(\text{copper})} &= \frac{\text{mass}(\text{copper}) \times C(\text{copper})}{C(\text{water})} \\ &= \frac{0.5 \text{ kg} \times 0.388 \text{ kJ/kgK}}{4.183 \text{ kJ/kgK}} \\ &= \mathbf{0.0464 \text{ kg (Ans.)}}\end{aligned}$$

b) Using the water equivalent in place of copper, the calorimeter consists of:

$$0.0464 + 0.6 \text{ kg of water} = 0.6464 \text{ kg of water}$$

∴ Heat absorbed by water + calorimeter

$$\begin{aligned}&= m C_{(\text{water})} \times \Delta T \\ &= 0.6464 \times 4.183 \times (31 - 22) \\ &= 0.6464 \times 4.183 \times 9 \text{ kJ} \\ &= \mathbf{24.33 \text{ kJ (Ans.)}}\end{aligned}$$

19. 0.5 kg of lead, at 51°C, is placed into a calorimeter containing 0.25 kg of water at 13.5°C. The final temperature of the calorimeter is 15.5°C. If the test determines the specific heat of the lead to be 0.1278 kJ/kgK, what is the water equivalent of the calorimeter?

$$\begin{aligned}\text{Heat lost by the lead} &= m c \Delta T \\ &= 0.5 \times 0.1278 \times (51 - 15.5) \\ &= 0.5 \times 0.1278 \times 35.5 \text{ kJ} \\ &= 2.268 \text{ kJ}\end{aligned}$$

Assuming all this heat is absorbed by water:

$$\begin{aligned}\text{Heat gained by water} &= m c \Delta T \\ 2.268 \text{ kJ} &= m \times 4.183 \times (15.5 - 13.5) \\ 2.268 &= m \times 4.183 \times 2 \\ m &= \frac{2.268}{4.183 \times 2} \\ &= 0.27 \text{ kg}\end{aligned}$$

But actual water in calorimeter = 0.25 kg

∴ Water equivalent of calorimeter

$$\begin{aligned}&= 0.27 - 0.25 \\ &= \mathbf{0.02 \text{ kg (Ans.)}}\end{aligned}$$